Period 3, Jan 27, 2025

41, ~~4~~, 44, 42, 43, ~~4~~, ~~10~~, ~~28~~, 40, ~~37~~

4.

$$\int \frac{x^2-1}{x^{3/2}}\,dx = \frac{2(x^2+3)}{3\sqrt{x}}+C$$

$$2-\frac{3}{2}=\frac{4}{2}-\frac{3}{2}=\frac{1}{2}$$

$$\frac{3}{2}+\frac{1}{2}=\frac{4}{2}=2$$

$$\int \frac{x^2}{x^{3/2}}\,dx-\int\frac{1}{x^{3/2}}\,dx=\int x^{\frac{1}{2}}\,dx-\int x^{-3/2}\,dx=1\cdot\frac{2}{3}\cdot x^{\frac{1}{2}+1=\frac{3}{2}}+1\cdot 2\cdot x^{-3/2+1=-\frac{1}{2}}+C$$

$$\frac{2x^{3/2}\sqrt{x}}{3\sqrt{x}}+\frac{3\cdot 2}{3\cdot\sqrt{x}}=\frac{2x^{\frac{3}{2}}\cdot x^{\frac{1}{2}}+6}{3\sqrt{x}}=\frac{2x^2+6}{3\sqrt{x}}=\frac{2(x^2+3)}{3\sqrt{x}}+C$$

10.

$$\int\frac{1}{4x^2}\,dx=\frac{1}{4}\int\frac{1}{x^2}\,dx=\frac{1}{4}\int x^{-2}\,dx=\frac{1}{4}\cdot\frac{1}{-1}\cdot x^{-2+1=-1}+C$$

$$-\frac{1}{4}x^{-1}+C=\frac{-1}{4x}+C$$

11.

$$\int\frac{1}{x\sqrt{x}}\,dx=\int\frac{1}{x^1\cdot x^{\frac{1}{2}}}\,dx=\int\frac{1}{x^{3/2}}\,dx=\int x^{-\frac{3}{2}}\,dx$$

$$1+\frac{1}{2}=\frac{2}{2}+\frac{1}{2}=\frac{3}{2}$$

$$1\cdot-\frac{2}{1}\cdot x^{-\frac{3}{2}+1=-\frac{1}{2}}+C$$

$$-2x^{-\frac{1}{2}}+C=\frac{-2\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}}+C=\frac{-2\sqrt{x}}{x}+C$$

28.

$$\int\frac{x^2+2x-3}{x^4}\,dx=\int\frac{x^2}{x^4}\,dx+\int\frac{2x}{x^4}\,dx-\int\frac{3}{x^4}\,dx=\int\frac{1}{x^2}\,dx+\int\frac{2}{x^3}\,dx-\int\frac{3}{x^4}\,dx$$

$$\int x^{-2}\,dx+\int 2x^{-3}\,dx-\int 3\cdot x^{-4}\,dx$$

$$1\cdot\frac{1}{-1}\cdot x^{-2+1=-1}+\cancel{2}\cdot\frac{1}{\cancel{-2}}\cdot x^{-3+1=-2}-\cancel{3}\cdot\frac{1}{\cancel{-3}}\cdot x^{-4+1=-3}+C=\frac{-1}{x^1}-\frac{1}{x^2}+\frac{1}{x^3}+C$$

37. $\int (1 - \csc T \cot T)\,dT = T + \csc T + C$

$\int 1\,dT = \int T^0\,dT = 1 \cdot \frac{1}{1} \cdot T^{0+1=1} + C = T$

40. $\int \sec y(\tan y - \sec y)\,dy = \int (\sec y \tan y - \sec^2 y)\,dy$

$\sec y - \tan y + C$

41. $\int (\tan^2 y + 1)\,dy$

$\int \sec^2 y\,dy = \tan y + C$

$\tan^2 y + 1 = \sec^2 y$

$\tan^2 y + 1$

$\frac{\sin^2 y}{\cos^2 y} + \frac{\cos^2 y}{\cos^2 y} = \frac{\sin^2 y + \cos^2 y}{\cos^2 y} = \frac{1}{\cos^2 y}$

$\sec^2 y$

42. $\int (4x - \csc^2 x)\,dx$

$4 \cdot \frac{1}{2} \cdot x^{1+1=2} + \cot x + C$

43 $\int \frac{\cos x}{1 - \cos^2 x}\,dx = \int \frac{\cos x}{\sin^2 x + \cancel{\cos^2 x} - \cancel{\cos^2 x}}\,dx$

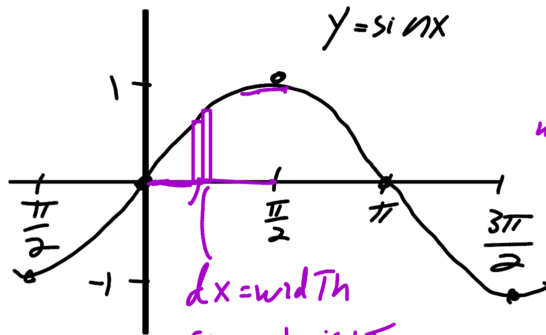
$\int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx = \int \cot x \csc x\,dx$

$-\csc x + C$

44. $\int \frac{\sin x \, dx}{1-\sin^2 x} = \int \frac{\sin x}{\cancel{\sin^2 x}+\cos^2 x-\cancel{\sin^2 x}} dx = \int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$

$\int \tan x \cdot \sec x \, dx$

$\sec x + c$



dx = width

sin x = height

width $= \frac{\pi}{n}$ where n = # of Rectangles

$\lim_{n\to\infty} \frac{\pi}{n} = dx = 0$

width of integral Rectangle

Summation

adding all Tiny Rectangles From 0 To π

$\int_0^{\pi} \sin x \cdot dx = -\cos x + c \Big[_0^{\pi} = [-\cos \pi + c] - [-\cos 0 + c]$

height width

$[-(-1)+c] - [-1+c]$

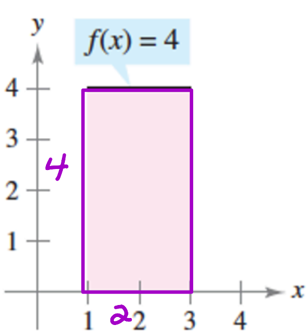
$1 + \cancel{c} + 1 - \cancel{c} = 2$

$f(x) = 4$

$$\int_1^3 4dx = 4x \Big[_1^3 = 4(3)+c-[4(1)+c]$$

$12 + \cancel{c} - 4 - \cancel{c} = 8$

Geometrical shape? Rectangle!



$A = bh$

$A = (3-1)(4)$

$A = 8$

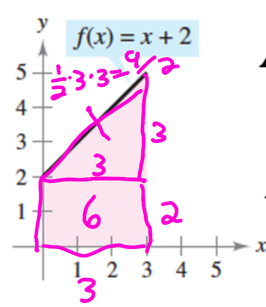
$2 \cdot 4 = 8$

$f(x) = x+2$

$$\int_0^3 (x+2)dx = \frac{1}{2}x^2+2x+c \Big[_0^3$$

$10\frac{1}{2} = \frac{21}{2} = \frac{9}{2}+\frac{12}{2} = \frac{9}{2}+6 = \frac{1}{2}(3)^2+2(3)+\cancel{c} - \cancel{[\frac{1}{2}(0)^2+2(0)+c]}$

Geometrical shape? Trapezoid!



$\frac{9}{2}+6 = \frac{21}{2}$

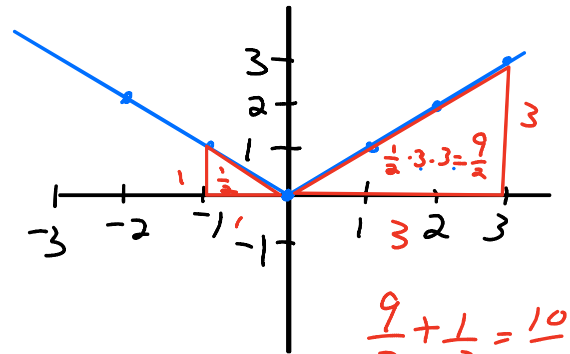
$A = \frac{1}{2}(b_1+b_2)h$

$A = \frac{1}{2}(2+5)3$

$= \frac{21}{2}$

Student Example 1: Evaluate the integral $\int_{-1}^{3} |x|dx$

$|F(x)| = 0$



x	$y = \|x\|$
0	0
1	1
-1	1
2	2
-2	2
3	3
-3	3

$\frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$

Example 3: Consider the function $f(x) = 3 - x$. Sketch a graph of this function.

$3 - 8 = -5$

A) What is the area between the curve and the x-axis between x = 4 and x = 8?

$\frac{1}{2}(-1 + -5) \cdot 4 = \frac{1}{2}(-6)(4)$

-12

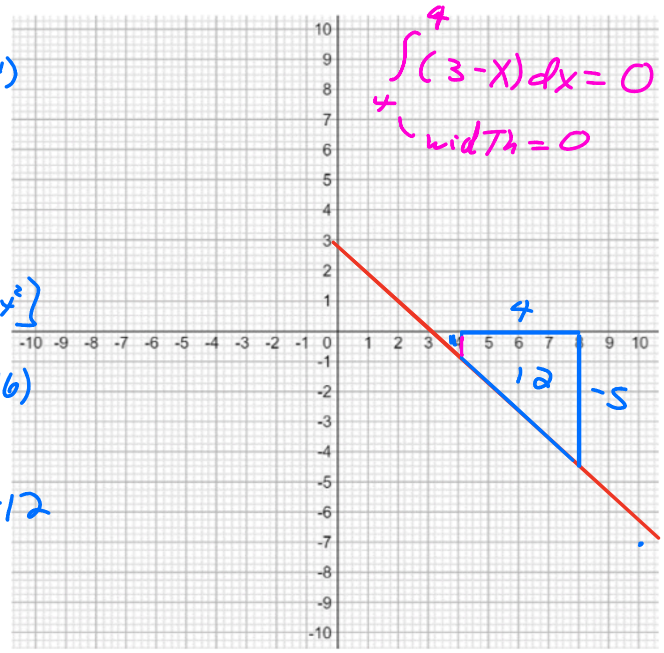
$\int_4^8 (3-x)dx = 3x - \frac{1}{2}x^2 + c \Big|_4^8$

$3(8) - \frac{1}{2}(8)^2 - [3(4) - \frac{1}{2}4^2]$

$24 - \frac{1}{2} \cdot 64 - 12 + \frac{1}{2}(16)$

$24 - 32 - 12 + 8$

$\cancel{32} \cancel{32} - 12 = -12$



$\int_4^4 (3-x)dx = 0$

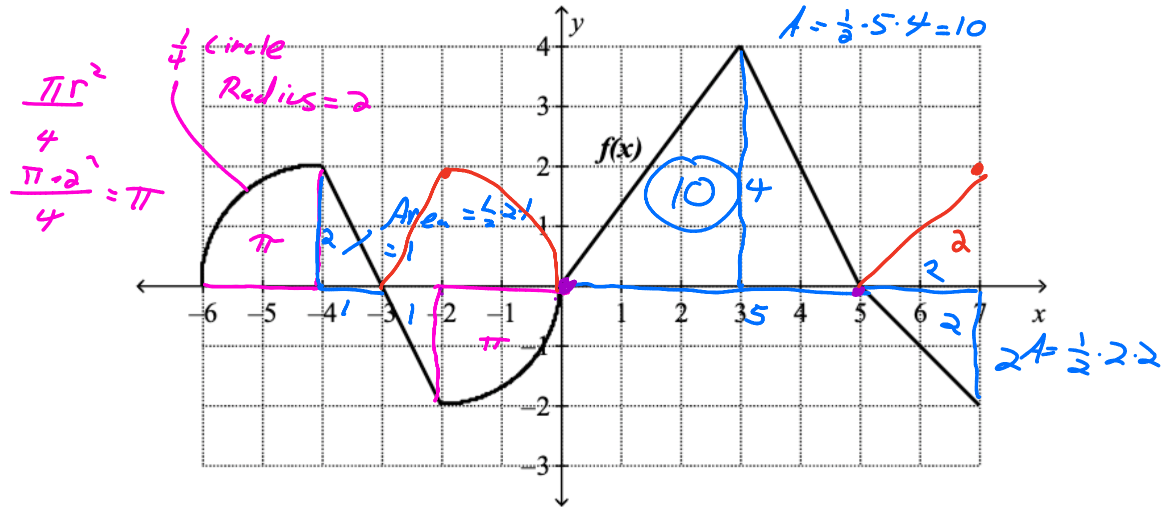
width $= 0$

b) $\int_4^8 f(x)\,dx$

$\int_8^4 (3-x)dx = 3x - \frac{1}{2}x^2 + c \Big|_8^4 = (3 \cdot 4 - \frac{1}{2}(4)^2) - (3 \cdot 8 - \frac{1}{2} \cdot 8^2)]$

$12 - 8 + -24 + 32 = 12 - 32 + 32 = 12$

Example Set 4: Use the graph above to find the indicated integrals.



a) $\int_0^5 f(x)dx = 10$ b) $\int_5^0 f(x)dx = -10$ c) $\int_{-2}^7 f(x)dx$

$\int_{-2}^{0} F(x)\,dx + \int_{0}^{5} F(x)\,dx + \int_{5}^{7} F(x)\,dx$

$-\pi + 10 + -2$

$-\pi + 8$

Use the graph above to find the indicated integrals.

d) $\int_{-6}^5 f(x)dx$

$\pi + 1 - 1 - \pi + 10$

10

e) $\int_{-6}^5 [f(x) + 2]\,dx$

$\int_{-6}^{5} F(x)\,dx + \int_{-6}^{5} 2\,dx$

$10 + 2x + c \Big[_{-6}^{5}$

$2(5) + c - [2(-6) + c]$

$10 + \cancel{c} + 12 \cancel{c}$

$10 + 22$

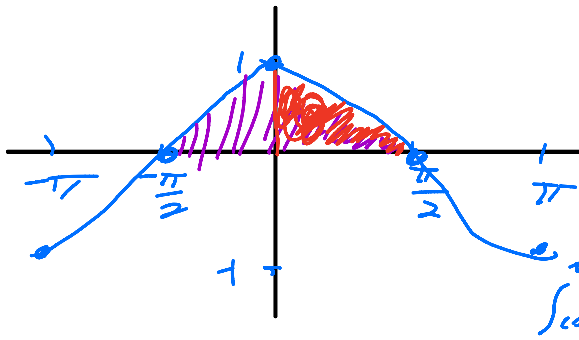
32

f) $\int_{-6}^7 |f(x)|\,dx$

$\pi + 1 + 1 + \pi + 10 + 2$

$14 + 2\pi$

$\int_{7}^{-6} |F(x)|\,dx = -14 - 2\pi$



$y = \cos x$

$\int_{-\pi/2}^{\pi/2} \cos x\,dx$

$\int_{-\pi/2}^{\pi/2} \cos x\,dx = 2 \cdot \int_{0}^{\pi/2} \cos x\,dx = 2 \cdot 1 = 2$

$\sin x + C \Big|_0^{\frac{\pi}{2}}$

$\sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$

In the AP Exam you will encounter questions of the following type:

1) Write a **definite integral** equal to the following limit:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^3 \frac{2}{n} \leftarrow \text{spread}$$

$$= \lim_{n\to\infty} \left[\left(1+\frac{2\cdot 1}{n}\right)^3 \cdot \frac{2}{n} + \left(1+\frac{2\cdot 2}{n}\right)^3 \cdot \frac{2}{n} + \left(1+\frac{3\cdot 2}{n}\right)^3 \frac{2}{n} + \dots \left(1+\frac{2(n-1)}{n}\right)^3 \frac{2}{n} + \left(1+\frac{2n}{n}\right)^3 \frac{2}{n} \right]$$

(A) $\cancel{\frac{1}{2}\int_1^2 x^3\,dx}$ (B) $\int_2^4 x^3\,dx$ (C) $\int_1^3 x^3\,dx$ (D) $\cancel{\int_0^1 (1 + x^3)\,dx}$

$$\lim_{n\to\infty} \frac{2}{n}\left[\left(1+\frac{2}{n}\right)^3 + \left(1+\frac{4}{n}\right)^3 + \left(1+\frac{6}{n}\right)^3 + \dots \left(1+\frac{2n-2}{n}\right)^3 + \left(1+\frac{2n}{n}\right)^3\right]$$

$$\frac{2}{n}\left[1^3 \dots \dots 3^3\right]$$